

f_B and the Heavy-light Spectrum from NRQCD

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The present status of lattice calculations of the B spectrum and f_B , using NRQCD for the b quark, is discussed.

1. INTRODUCTION

B physics is a subject of active research. The theoretical and experimental understanding of the B meson and b baryon spectrum is just beginning. Weak matrix elements of B mesons, e.g. f_B , f_{B_s} , B_B , and B_{B_s} are being studied to determine fundamental parameters of the Standard Model. This review summarizes the progress made in calculating the B spectrum and decay constants using the nonrelativistic QCD (NRQCD) [1] approach on the lattice.

The advantage of NRQCD is that the rest mass term is removed from the Lagrangian. Hence, large $O(Ma)$ effects pose no problem, and one can simulate b quarks directly on the lattice. Alternate approaches have been reviewed by T. Onogi [2] at this conference.

In heavy-light mesons, NRQCD is equivalent to a $1/M$ expansion. The rationale is that if the heavy quark is nonrelativistic ($p = Mv$), and the light quark is relativistic with a momentum $p \sim \Lambda_{QCD}$, then momentum conservation in the meson rest frame gives

$$Mv \sim \Lambda_{QCD}. \quad (1)$$

For B mesons, one has:

$$\Lambda_{QCD}/M \sim 0.1. \quad (2)$$

It is thus reasonable to include relativistic corrections in an expansion in powers of Λ_{QCD}/M . The lowest terms in the expansion are as follows. The $1/M$ corrections to the static NRQCD Lagrangian density $L = D_t$ are

$$H^{(1)} = -\frac{\vec{D}^2}{2M_0} - \frac{g}{2M_0} \vec{\sigma} \vec{B}, \quad (3)$$

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NRQCD action	group	light quark action	β	V
quenched lattices				
$1/M$	Hiroshima	Wilson	5.8	$16^3 \times 32$
	Draper <i>et al.</i>	Wilson	6.0	$20^3 \times 32$
	SGO	clover	6.0	$16^3 \times 48$
$1/M^2$	Hiroshima	Wilson	5.8	$16^3 \times 32$
	GLOK	clover	5.7	$12^3 \times 24$
	GLOK	clover	6.0	$16^3 \times 48$
$n_f = 2$ staggered lattices from HEMCGC				
$1/M$	SGO	Wilson	5.6	$16^3 \times 48$
	SGO	clover	5.6	$16^3 \times 48$

Table 1
NRQCD calculations of the B spectrum and decay constants. For references see text.

and the $1/M^2$ corrections are

$$H^{(2)} = \frac{ig}{8M_0^2} (\vec{D}\vec{E} - \vec{E}\vec{D}) - \frac{g}{8M_0^2} \vec{\sigma} (\vec{D} \times \vec{E} - \vec{E} \times \vec{D}) - \frac{(\vec{D}^2)^2}{8M_0^3}. \quad (4)$$

In the following, actions which only include $H^{(1)}$ will be referred to as $1/M$ actions, and those which include $H^{(2)}$, as $1/M^2$ actions. Note that $H^{(2)}$ also includes the first relativistic correction to the kinetic energy which is formally $O(1/M^3)$, but is expected to give a contribution of a similar size as the spin-dependent interactions $O(1/M^2)$. An overview of the simulations covered is given in Table 1. For lack of space I have not included a recent calculation on coarse lattices using improved glue and light fermions [3].

In this review, I will make comparisons to ex-

periment and to other lattice results where possible. To judge the reliability of the predictions, I also evaluate systematic effects like the effect of truncating the $1/M$ expansion, quenching errors, and the dependence of the decay constant on lattice spacing.

2. SPECTRUM

Using nonrelativistic b quarks and clover light quarks on the lattice, it is possible to reproduce the presently known general features of the B spectrum. An overview of the the meson spectrum at a lattice spacing $a^{-1} \sim 2$ GeV is presented in Fig. 1. The plot shows results from the most comprehensive calculations on quenched ($1/M^2$ action, GLOK [4]) and dynamical ($1/M$ action, SGO [5]) configurations. The $B_s - B_d$ splitting agrees with experiment, also the $B(2S)$ agrees well with the first experimental candidate [6].

For the P wave states, two kinds of experimental signals have been found. A $B^{(*)}\pi$ resonance has been established which is expected to be a superposition of various P states. The states with light quark angular momentum $j_l = 1/2$ (B_0^* and B_1^*) and $j_l = 3/2$ ($B_1'^*$ and B_2^*) are expected to form doublets. The splittings within these doublets are given by the coupling of the heavy quark spin to j_l . The second experimental signal is a narrow $B\pi\pi$ resonance which has probably $j_l = 3/2$ [6].

Lattice NRQCD predicts a $B_2^* - B_0^*$ splitting of the order of 200 MeV (an effect of $\sim 4\sigma$). The B_1^* and $B_1'^*$ cannot yet be resolved since each of the two operators used in the simulation for the $j = 1$ states project only on a superposition of these states, called \overline{B}_1^* in Fig. 1. For $n_f = 2$ lattices, there is presently only a P state signal in the channel that (similar to the quenched data point on the left of it) corresponds to the 1P_1 state in heavy-heavy mesons. Given the limitations that apply to both the experimental and lattice results on P states, my conclusion is that there is qualitative agreement.

A study of the effect of the $1/M^2$ terms in the action on some spectral quantities is ongoing [7]. A low statistics comparison between $1/M$

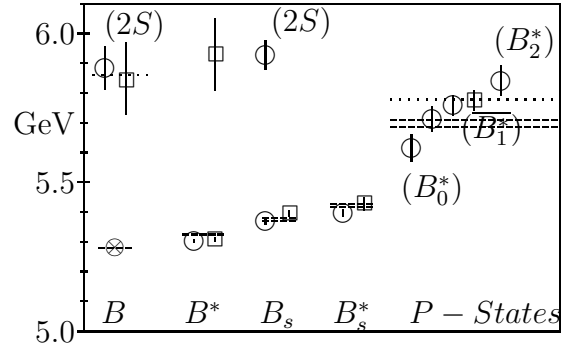


Figure 1. The B meson spectrum from lattice NRQCD. Circles denote quenched results, squares, results from $n_f = 2$ lattices. Dashed lines denote experimental error bounds as given by the Particle Data Book, dotted lines the central values of first experimental measurements of these states.

and $1/M^2$ actions on quenched configurations at $\beta = 6.0$ [5,4], suggests that there is no significant difference. Assuming that this is true, a comparison of the spectrum on quenched and the $n_f = 2$ lattices from HEMCGC, as shown in Fig. 1, further suggests that quenching errors are small too.

Lastly, it is important to study the dependence of the results on the lattice spacing a . Preliminary results of a comparison between quenched lattices at $\beta = 5.7$ and 6.0 show no significant scaling violations [8].

2.1. The Hyperfine Splitting

The $B^* - B$ splitting is of particular interest since it is expected to be sensitive to various effects such as discretization errors, quenching and the tuning of the parameters in the heavy quark action. The results shown in Fig. 2 are ~ 20 MeV too low. The question arises whether the results improve for smaller a . Fig. 2 shows results using Wilson and clover light quarks at various lattice spacings, with a determined from m_ρ . With present errors, I conclude that there is reasonable scaling for clover light quark data at $\beta = 5.7$ and 6.0 , as well as for Wilson light quark data at $\beta = 5.8$ and 6.0 . Also, there is no obvious

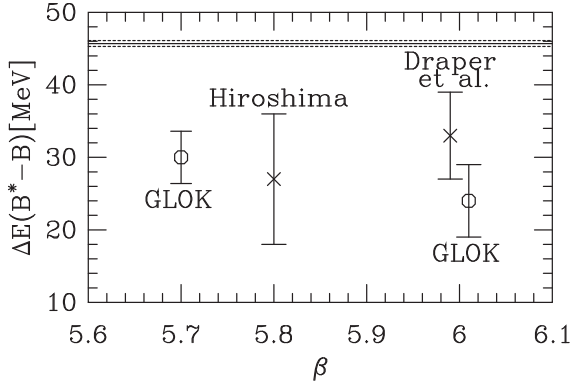


Figure 2. NRQCD results for the B hyperfine splitting. Crosses use Wilson light quarks, circles tadpole-improved clover light quarks. The solid and dotted lines denote the experimental value with its error bounds.

difference between clover and Wilson results.

If the B hyperfine splitting depends on the heavy quark wavefunction at the origin, one would expect quenching effects to be significant. Comparing results on quenched and dynamical ($n_f = 2$) configurations (Refs. [5,4], does not show any significant effect. One possibility is that the physical picture of a nonrelativistic wavefunction is incorrect in B mesons. However, the sea quark mass in the HEMCGC configurations might be too large, so modern dynamical simulations with high statistics are needed to settle this issue. Another quantity whose effect on the hyperfine splitting needs to be studied are the coefficients of the spin-dependent terms in the action. A perturbative calculation by H. Trotter [9] is under way.

2.2. Comparison to other lattice calculations

Fig. 3 shows data from NRQCD and other lattice methods along with a preliminary experimental result for a P state excited B_s meson. In some of the calculations only an approximate value for the strange quark mass is used [10,11]. Also, the calculations were done at slightly different β values between 5.9 and 6.1. (The lattice spacing was set using either m_ρ with clover quarks or $\sqrt{\sigma}$. These give roughly consistent values for a fixed

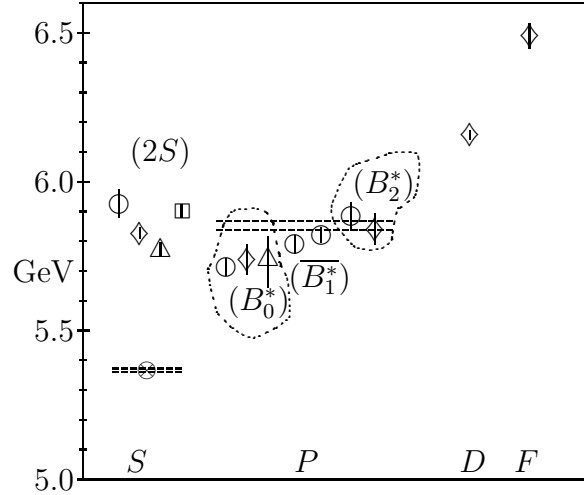


Figure 3. Compilation of lattice results for excited B_s mesons. Circles are NRQCD results, diamonds [10] and triangles [11] show static heavy quarks. The square shows tadpole-improved clover heavy quarks [12]. Results for the B_0^* and the B_2^* are grouped together with dotted circles. The meaning of the dashed lines is the same as in Fig. 1. At the bottom of the figure, the orbital quantum numbers of the respective states are shown.

β in this β range.) Because of these limitations, a check of the expected $1/M$ corrections ($O(50)$ MeV) to the static limit cannot be made with these data. Lastly, I would like to mention that first results for D and F wave states are reported in Ref. [10].

2.3. b Baryons

Lattice results for b baryons with non-static b quarks are summarized in Fig. 4. For the Λ_b the quenched lattice calculations using NRQCD [4], tree-level clover [13], and plain Wilson [14] heavy quarks agree within errors with experiment and with each other. The Λ_b from Ref. [5] is significantly heavier than the experimental value and heavier than the quenched NRQCD result. This needs further study.

For the Σ_b and Σ_b^* baryons there are only pre-

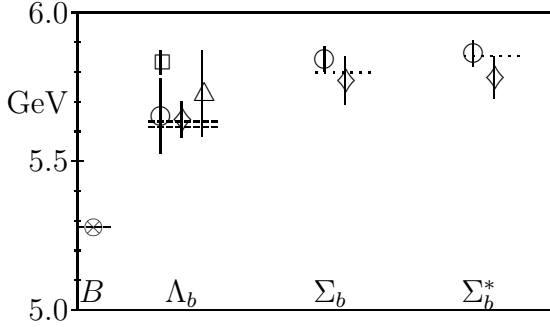


Figure 4. Lattice results on baryons containing one b and two light quarks. Only lattice calculations with moving (non-static) b quarks have been considered. Circles denote NRQCD b quarks [4], diamonds show clover ($c_{SW} = 1$) heavy quarks [13], and the triangle, Wilson heavy quarks [14]. The square is from a SGO NRQCD calculation with $n_f = 2$ [5]. The meaning of the lines is the same as in Fig. 1.

liminary experimental results available [15], but they are in agreement with the available lattice results (see Fig. 4). Ref. [13] also has results for strange b baryons.

3. DECAY CONSTANTS

At tree level, the nonrelativistic current, correct through $O(1/M^2)$, is given by:

$$A_0 = \bar{q}\gamma_0\gamma_5 Q \quad (5)$$

$$- \frac{1}{M_0} \bar{q}\gamma_0\gamma_5 \left(\vec{\gamma}\vec{D} \right) Q \quad (6)$$

$$+ \frac{1}{8M_0^2} \bar{q}\gamma_0\gamma_5 \left(\vec{D}^2 + g\vec{\Sigma}\vec{B} - 2i\vec{\alpha}\vec{E} \right) Q, \quad (7)$$

where $\vec{\alpha} = \gamma_0\vec{\gamma}$.

The question is whether the truncation of the $1/M$ expansion at this order is sensible. At present, the effects of including both $1/M$ and $1/M^2$ corrections have been studied in two calculations: Ref. [16] with Wilson light quarks and Ref. [4] with clover light quarks. First I examine the effect of the $1/M^2$ terms in the action on the matrix elements of the zeroth order cur-

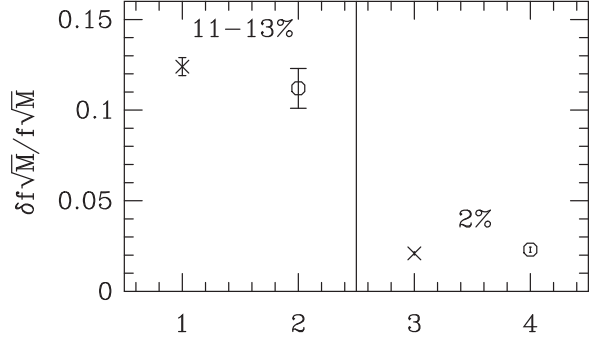


Figure 5. Ratio of the unrenormalized current corrections (left, $O(1/M)$, right, $O(1/M^2)$) to the uncorrected current. Crosses use Wilson light quarks at $\beta = 5.8$ [16], circles, clover light quarks at $\beta = 6.0$ (GLOK). For the x axis, an arbitrary numbering is used.

rent (Eq. (5)): Comparing the $1/M$ and $1/M^2$ actions with clover light quarks at $\beta = 6.0$, one finds a difference of $\sim 7\%$ [4,17]. However, this is only a $\sim 2\sigma$ effect as the results from [17] are low statistics. Similarly, the Hiroshima group finds the effect of including the $1/M^2$ corrections to be $< 0.5\%$ [16].

Next the effect of the $1/M^2$ current corrections is studied. Fig. 5 shows ratios of the $1/M$ and $1/M^2$ current corrections to the zeroth order current (Eq. 5) at the mass of the B meson. The ratio of the $O(1/M)$ operator is $\sim 10\%$, and of the $O(1/M^2)$ is $\sim 2\%$. After including renormalization factors in the ratios, the $1/M$ correction is still $\sim 10\%$ [18], whereas the renormalization constant for the $1/M^2$ current corrections has not been calculated yet.

The renormalized matrix elements are obtained from a matching calculation to full QCD. For NRQCD heavy quarks, two such 1-loop calculations for operators through $1/M$ exist. In Ref. [19] the full operator mixing at $O(\alpha)$ has been calculated for clover light quarks. This calculation shows that there is a significant mixing with an $O(\alpha\Lambda_{QCD}a)$ current correction. Ref. [20] presents a calculation for Wilson light quarks which takes only the diagrams with vanishing external momenta into account. These give only the

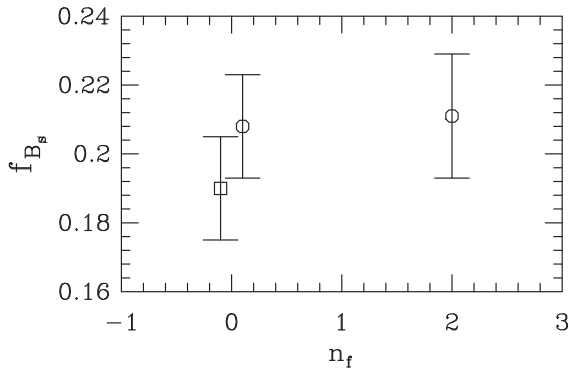


Figure 6. Comparison of f_{B_s} from NRQCD, using configurations with $n_f = 0$ and $n_f = 2$ [5]. Quenched results using a $1/M$ action [17] (circle) and a $1/M^2$ action [4] (square) are shown. The lattice spacing has been fixed from m_ρ ($a^{-1} \simeq 2$ GeV). For better comparison, the $1/M^2$ current corrections are not included in the result with the $1/M^2$ action.

contributions of Eq. (5) and Eq. (6) on the renormalization of the zeroth order current Eq. (5). The neglected terms are however significant [18]. Also note that for Wilson light quarks the full mixing calculation at one loop contains an infrared divergence unless the $O(\alpha\Lambda_{QCD}a)$ correction is neglected.

In Table 2, a summary of f_B is given; NRQCD data includes renormalization constants. I give two results for the Hiroshima group: (i) from Ref. [21], using the renormalization constant determined by them, and (ii) using the full perturbative Z 's (but neglecting any $O(\alpha\Lambda_{QCD}a)$ terms) at $O(1/M)$ [19] on their raw data from Ref. [16]. To obtain a renormalized value for f_B from the raw data provided by Draper *et al.* [22], I also use the renormalization constant from [19]. However, note that these Z 's are for a slightly different action than used in Refs. [16,22].

The first error bar in the NRQCD results in Table 2 is statistical (including fitting uncertainties where applicable). The second error estimates the uncertainty in the determination of a^{-1} . For clover light quarks, the upper error bound comes from determining a^{-1} from f_π ; the lower bound

from either $\sqrt{\sigma}$ [4], or from the statistical error in m_ρ [5]. Ref. [17] used an a^{-1} of 2 GeV with an error of 0.2 GeV. For Wilson light quarks, the upper error bound comes from the error in a^{-1} from m_ρ , the lower error bound from the determination of a^{-1} from $\sqrt{\sigma}$. Here, the difference between a^{-1} from m_ρ and $\sqrt{\sigma}$ at $\beta = 6.0$ is $\sim 20\%$, whereas for the clover action they are roughly consistent. The third error bar is an estimate of higher order contributions in perturbation theory, obtained by using α_P [23] evaluated at $q^* = 1/a$ and at $q^* = \pi/a$ in the Z 's, and, for calculations including only $O(1/M)$ corrections it includes an estimate of the $1/M^2$ contributions.

Table 2 also lists results on f_B from relativistic heavy quark actions. For more details on this data see the review by T. Onogi [2]. It is encouraging to note that the result with NRQCD heavy and clover light quarks [4] agrees well with those results with relativistic heavy quarks where extrapolations to $a \rightarrow 0$ have been done [25–27]. The results with NRQCD heavy and Wilson light quarks [16,22] in Table 2 are high (> 200 MeV). This is mainly due to the choice of the lattice scale; using a^{-1} from $\sqrt{\sigma}$ instead of Wilson m_ρ yields a result close to [4,25–27]. Wilson light quarks give rise to considerable discretization effects $O(\Lambda_{QCD})$, and if they are used in conjunction with NRQCD heavy quarks, these effects cannot be removed by extrapolating to $a \rightarrow 0$ (which is in principle possible e.g. in [27]).

Since f_B from NRQCD cannot be extrapolated to $a \rightarrow 0$, it is important to understand the discretization effects. There exist only preliminary results for f_{B_s} as reported by J. Hein *et al.* [8]. They find slight scaling violations: f_{B_s} decreases by $\sim 2\sigma$ between $\beta = 5.7$ and 6.0. To make a clear statement about discretization effects, at least one other calculation at smaller a is required.

Another important question is how quenching affects the heavy-light decay constant. In Fig. 6, f_{B_s} from two quenched calculations [17,4] is compared with a result using $n_f = 2$ lattices [5]. To accommodate the fact that the strange quark masses in the two quenched calculations have been determined by either setting K or K^* mesons to their physical value, the systematic error from various methods to fix the strange

Table 2

Summary of results on f_B from moving (non-static) heavy quarks. For the results marked with *, I have included the renormalization constants from Ref. [19]. “t.i. clover” stands for tadpole-improved clover. The results from Refs. [16,22,4,5,26] and the systematic errors from [25] are preliminary.

group	heavy quark	light quark	β	a^{-1} from	f_B [MeV]
NRQCD heavy quarks, quenched lattices					
Hiroshima [21]	NRQCD	Wilson	5.8	m_ρ	202(17)($^{+11}_{-68}$)(18) 210(8)($^{+11}_{-71}$)(6)*
Draper <i>et al.</i> [22]	NRQCD	Wilson	6.0	m_ρ	232(11)($^{+8}_{-64}$)(6)*
SGO [17]	NRQCD	t.i. clover	6.0	m_ρ	183(30)(28)(8)
GLOK [4]	NRQCD	t.i. clover	6.0	m_ρ	149(12)($^{+22}_{-5}$)(9)
NRQCD heavy quarks, $n_f = 2$ lattices from HEMCGC					
SGO [5]	NRQCD	t.i. clover	5.6	m_ρ	156(4)($^{+36}_{-4}$)(11)
Relativistic heavy quark actions, quenched lattices					
APE [24]	clover ($c_{SW}=1$)	clover ($c_{SW}=1$)	6.2	f_π	180(32)
Fermilab [25]	t.i. clover	t.i. clover	5.7, 5.9, 6.1	f_K	156(35)
JLQCD [26]	t.i. clover	t.i. clover	5.9, 6.1, 6.3	m_ρ	163($^{+18}_{-20}$)
MILC [27]	Wilson	Wilson	5.7, 5.85, 6.0, 6.3, 6.52	f_π	153($^{+40}_{-16}$)
UKQCD [28]	clover ($c_{SW}=1$)	clover ($c_{SW}=1$)	6.2	$m_\rho, \sqrt{\sigma}$	160($^{+53}_{-20}$)

quark mass have been included in the error bars. Existing data in Fig. 6 show no significant deviation between the quenched and the dynamical results.

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